Name: ____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** The probability function $P : \Omega \to \mathbb{R}$ gives the probability of any single outcome occurring.

Solution: The domain of the probability function is subsets of Ω , not just Ω .

2. True **FALSE** Let X, Y be random variables such that X + Y = 1. Then X and Y can be independent variables.

Solution: They can't be independent because once we know what X is, we know exactly what Y is.

Show your work and justify your answers. Please circle or box your final answer.

- 3. (10 points) I have a loaded coin such that head appears three times as likely as tails. Suppose I get paid 1 dollar for every head that I flip. I flip the coin once and want to know how much I get paid.
 - (a) (4 points) Describe what the triplet (Ω, P, X) is in this scenario.

Solution: $\Omega = \{H, T\}, P : \{\emptyset, \{H\}, \{T\}, \{H, T\}\} \to [0, 1] \text{ by } P(\emptyset) = 0, P(\Omega) = 1, P(\{H\}) = 3/4, P(\{T\}) = 1/4.$ Finally $X : \Omega \to \mathbb{R}$ is 1 if heads, 0 if tails.

(b) (5 points) Graph any of the following that apply (be sure to clearly denote what you are drawing): PDF, PMF, CDF

Solution: The PMF applies. It only has a stalk at 0 and 1 with a stalk of height $\frac{1}{4}$ at 0 and stalk of height $\frac{3}{4}$ at 1. The CDF also applies. It looks like a step function

$$F(x) = \begin{cases} 0 & x < 0\\ \frac{1}{4} & 0 \le x < 1\\ 1 & 1 \le x \end{cases}$$

The PDF does not apply because this is a discrete random variable..

(c) (1 point) Calculate the payout and the standard error of this game.

Solution: The payout is $\frac{1}{4} \cdot 0 + \frac{3}{4} \cdot 1 = \frac{3}{4}$. The standard error is $E((X - \mu)^2) = \frac{1}{4} \cdot (0 - 3/4)^2 + \frac{3}{4} \cdot (1 - 3/4)^2 = \frac{3}{16}$.